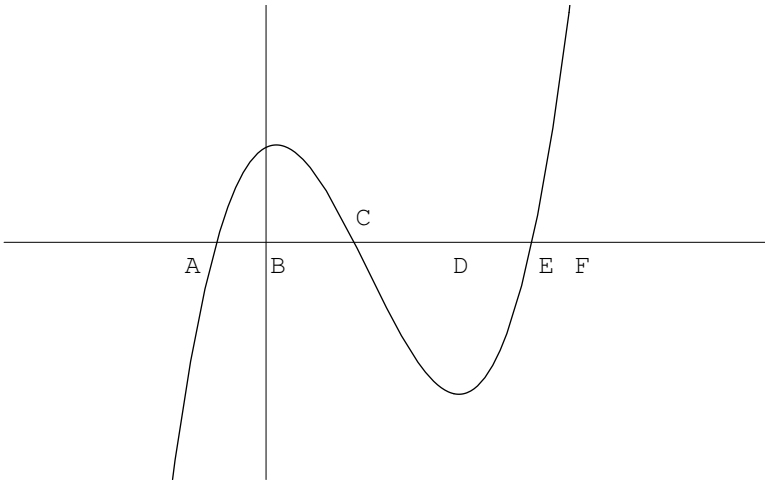


*You may use your calculators on this test ... but make sure you show your mathematics!*

1. Find the critical numbers of  $f(x) = x^{4/3} - x^{1/3}$ .
2. Find the critical numbers of  $f(x) = x^{1/3}(x + 4)$ .
3. Find the critical numbers of  $f(x) = (x - 3)(x - 5)^2$ .
4. Find the critical numbers of  $f(x) = \begin{cases} 25 - x & \text{for } x \leq 5 \\ x^2 - 5 & \text{for } x > 5 \end{cases}$ .
5. Find the critical numbers of  $f(x) = \begin{cases} 16 - x & \text{for } x \geq 3 \\ x^2 - 3 & \text{for } x < 3 \end{cases}$ .
6. Find the relative minimum of  $f(x) = (x - 3)(x - 4)^2$ .
7. Find the relative maximum of  $f(x) = (x - 7)^2(x + 3)$ .
8. Find the relative maximum of  $f(x) = x^2\sqrt{9 - x^2}$ .
9. If  $f(x) = 4x^2 + \frac{k}{x}$  has a relative extrema at  $x = 1$ , find the value of  $k$ .
10. If  $f(x) = 4x^2 + \frac{k}{x}$  has an inflection point at  $x = -1$ , find the value of  $k$ .
11. For what values of  $x$  is  $f(x) = \frac{7}{5 - x}$  concave down?
12. For what values of  $x$  is  $f(x) = \frac{3}{x + 2}$  concave up?
13. For what values of  $x$  is  $f(x) = x^2e^x$  concave down?
14. Find the absolute maximum of  $f(x) = 5 - 6x^2 - 2x^3$  on  $[-3, 1]$ .
15. Find the absolute minimum of  $g(x) = x^4 - 5x^2 + 4$  on  $[0, 2]$ .
16. Find the absolute maximum of  $h(x) = 1 - x^{2/3}$  on  $[-1, 8]$ .
17. Find all the values of  $c$  that satisfy the Mean Value Theorem for  $f(x) = x^2 + x$  on  $[-4, 6]$ . You may assume the MVT holds for  $f$  on the given interval.
18. Find all the values of  $c$  that satisfy the Mean Value Theorem for  $f(x) = x^3 - 3x^2 + 2x$  on  $[0, 2]$ . You may assume the MVT holds for  $f$  on the given interval.
19. Find the inflection points (if any) of  $f(x) = x^3 - x^2 + x + 1$ .
20. Find the inflection points (if any) of  $2 \cos 2x$  on  $(0, \pi)$ .
21. Use the Second Derivative Test to find the relative extrema of  $f(x) = x^3 - 5x - 2$ .
22. Use the Second Derivative Test to find the relative extrema of  $f(x) = x^4 - 2x^2 + 7$ .

23. The graph of  $f'$  is given below.

Determine where  $f$  has relative maximums and minimums then state the intervals where  $f$  is concave up or concave down. (Ignore the "F")



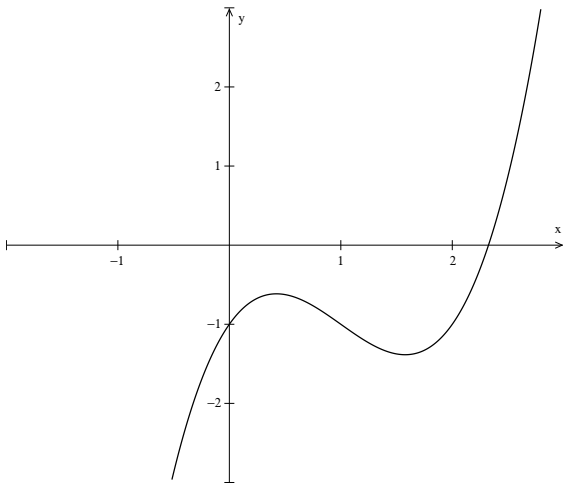
24. The graph of  $f$  is given below.

Label "A" where  $f'(x) > 0$  and  $f''(x) > 0$ .

Label "B" where  $f'(x) > 0$  and  $f''(x) < 0$ .

Label "C" where  $f'(x) < 0$  and  $f''(x) > 0$ .

Label "D" where  $f'(x) < 0$  and  $f''(x) < 0$ .



Answers (not complete solutions)

1.  $x = \frac{1}{4}$  and  $x = 0$
2.  $x = -1$  and  $x = 0$
3.  $x = \frac{11}{3}$  and  $x = 5$
4.  $x = 5$
5.  $x = 0$  and  $x = 3$
6. Relative minimum of 0 at  $x = 4$
7. Relative maximum of 148.148 at  $x = .333$
8. Relative maximum of 10.392 at  $x = -2.449$  and relative maximum of 10.392 at  $x = 2.449$
9.  $k = 8$
10.  $k = 4$
11.  $(5, \infty)$
12.  $(-2, \infty)$
13.  $(-3.414, -.586)$
14. Absolute maximum of 5 at  $x = -3$  and  $x = 0$
15. Absolute minimum of -2.250 at  $x = 1.581$
16. Absolute maximum of 1 at  $x = 0$
17.  $c = 1$
18.  $c = .423$  or  $c = 1.557$
19.  $(.333, 1.259)$
20.  $(\frac{\pi}{4}, 0)$  and  $(\frac{3\pi}{4}, 0)$
21. Relative maximum of 2.303 at  $x = -1.291$   
Relative minimum of -6.303 at  $x = 1.291$
22. Relative minimum of 6 at  $x = -1$   
Relative minimum of 6 at  $x = 1$   
Relative maximum of 7 at  $x = 0$
23. Relative minimum "A" and "E"  
Relative maximum "C"  
Concave up on  $(-\infty, B) \cup (D, \infty)$   
Concave down on  $(B, D)$

24. Below.

